1 Lab 3 Question (2d) Solution

This is the solution Dr. Obimbo showed me. I have made annotations to it – hopefully you'll find it clear and useful. Please e-mail me if you have any questions about it.

The original question asks the student to prove by induction that $n! > n^2$ for $n > a, n \in N$. We demonstrate that $a = 3$ with a table of values, and continue the induction from there.

Note that $n > 3$ is the same as $n \geq 4, n \in N$ – this is useful in [IS].

[BC] Demonstrate $n! > n^2$ for $n = 4$.

$(4)! = 24 = \text{LHS} > \text{RHS} = (4)^2 = 16$

$\therefore$ LHS > RHS

[IH] Assume true for $n = k$.

$(k)! > (k)^2, k \geq 4$.

[IS] Try for $n = (k + 1)$.

$\text{RHS} = (k + 1)^2$

$\text{RHS} = k^2 + 2k + 1$

* [Since $k^2 > k$, and $k^2 > 1$, ok to substitute these terms with k^2.] *

$\therefore \text{RHS} = k^2 + 2(k^2) + (k^2)$

$\text{RHS} = 4k^2$

[From IH, we had assumed that $k! > k^2$.]

$\therefore 4k! > 4k^2$

* [But $k \geq 4$, ok to substitute $k + 1$ for 4.] *

$\therefore (k + 1)k! > 4k^2$

$\therefore (k + 1)k! = (k + 1)!$

[QED]

* The first tricky step is the substitution of k and 1 with k^2. The motivation here is that we want to get the coefficient 4 to appear. The justification is that we're preserving the inequality – that the RHS is still surely less than the LHS despite our changes.

* The second tricky step is the substitution of 4 with $k + 1$. This is the final step that finalizes linking n to $k+1$. The justification is again that we're preserving the inequality, since 4 is certainly less than $k + 1$.

Thanks,
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